MA 214 **Full name:** ____________________
1/31/2024
Quiz 2
Version A **Student ID number:** 9____________________

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1. (12 points) Solve the following ODE using the integrating factor method. Solve explicitly for y. You can assume t is always a positive number.

$$ty' - 2y = t^4 + 3t^3 \qquad (*)$$

1. Write $(*)$ in the basic form dividing both sides by "t":

$$y' - \frac{2}{t}y = t^3 + 3t^2$$

Compare to $y' + p(t)y = q(t)$, so $p(t) = -\frac{2}{t}$ and $q(t) = t^3 + 3t^2$

2. Compute the integrating factor $\mu(t)$:

$$\mu(t) = e^{\int p(t)dt} = e^{\int -\frac{2}{t}dt} = e^{-2\ln t} = \cancel{e^{\ln t^{-2}}} = t^{-2}$$

3. Multiply both sides by "$\mu(t)$" and integrate:

$$\mu(t)y' + \mu(t)p(t)y = \mu(t)q(t) \qquad (**)$$

but $\mu(t)p(t) = \mu'(t)$, so

$$\mu(t)y' + \mu'(t)y = \mu(t)q(t)$$

but $(\mu(t)y)' = \mu(t)y' + \mu'(t)y$ (product rule for deriv.)

$\Rightarrow (\mu(t)y)' = \mu(t)q(t)$ (by $(**)$)

$\Rightarrow \int(\mu(t)y)'dt = \int \mu(t)q(t)dt \Rightarrow \mu(t)y = \int \mu(t)q(t)dt$

$\int \mu(t)q(t)dt = \int t^{-2}(t^3 + 3t^2)dt = \int (t+3)dt = \frac{t^2}{2} + 3t + C$

$\Rightarrow t^{-2}y = \frac{t^2}{2} + 3t + C \Rightarrow y = t^2\left(\frac{t^2}{2} + 3t + C\right) = \boxed{\frac{t^4}{2} + 3t^3 + Ct^2}$

2. A pool initially contains 120 kg of chlorine dissolved in 850 gallons of water. A chlorine solution that has concentration 0.3 kg/gal pours into the pool at a rate of 4 gal/minute. The water in the pool is well-mixed, and 3 gal/minute of pool water are pumped out of the pool.

   (a) (3 points) How many gallons of salt water are in the pool at time t?

You are asked to write the equation for the volume of the water in the tank at time "t":

$$V(t) = \underbrace{850}_{\text{initial volume}} + (\underbrace{4}_{\dot{q}_{in}\text{: incoming flow rate}} - \underbrace{3}_{\dot{q}_{out}\text{: outgoing flow rate}})t = \boxed{850 + t}$$

   (b) (5 points) Set up an initial value problem to find the amount of chlorine $C(t)$, measured in kg, in the pool after t minutes. Your answer should include a differential equation and an initial condition. **Do not solve the differential equation.**

Here you need to set up the differential equation whose solution is $C(t)$: amount of chlorine.

$$\frac{dC}{dt} = \begin{matrix}\text{in flow rate} \\ \times \text{ in concentration}\end{matrix} - \begin{matrix}\text{out flow rate} \\ \times \text{ out concentration}\end{matrix}$$

$$= \dot{q}_{in} \cdot C_{in} - \dot{q}_{out} \cdot C_{out}$$

where $C_{out} = \dfrac{C(t)}{V(t)} = \dfrac{C(t)}{850+t}$

$$\Rightarrow \boxed{\frac{dC}{dt} = 4 \cdot 3 - 3 \cdot \frac{C(t)}{850+t}}$$